Accuracy analysis of the acoustic discharge measurement using analytical, spatial velocity profiles

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Introduction

One of the main issues of the acoustic discharge measurement (ADM) is related to the accuracy of the method, which depends on the local flow situation, the local geometry, the transducer installation, the electronic devices and the protrusion of the acoustic transducers into the flow. To estimate the overall accuracy of a measurement individual sources of errors have to be analyzed separately [1]. Important sources of errors arise from flow field distortions, such as spatially varying flow fields and cross flow in the measuring section. In practice multipath installation and crossed paths are used to increase measuring accuracy. The following paper focuses on errors due to flow field distortions: On the one hand the integration error arising from the numerical integration of the discharge using the measured velocities and on the other hand the error due to cross flow effects. For the analysis of these uncertainties spatially varying flow fields are investigated. In a first step analytical flow profiles without swirling components are used and in a next step rotation of the flow field is introduced. To demonstrate the effects of integration and cross flow errors, a simulated ADM measurement is discussed for a numerically simulated flow field downstream of two bends and upstream of a spiral casing.

1 Multipath acoustic discharge measurement

Figure 1 shows a typical 2x4-path application in a closed conduit. On each acoustic plane four paths are arranged. The two crossed planes are used to reduce the negative effect of cross flow in the measuring section.

In a closed conduit the two crossed planes with the acoustic paths, lie diagonally in the pipe. Using the transit time difference of an acoustic signal the averaged flow velocity projected on a path can be determined. From these measured velocities the axial velocity components can be estimated. The integration of the axial velocity components yields the discharge. Errors can occur in each of these steps leading to the determination of discharge. In the following the integration error and the cross flow error are discussed for spatially varying flow fields.
1.1 Integration error

Discharge can be determined by integrating the normal velocity component on any plane intersecting the pipe. For a plane perpendicular to the pipe axis the normal velocity component is equivalent to the axial velocity component. The perpendicular plane located at the intersection of the acoustic planes is called midplane.

\[ Q = \int_{-R}^{R} L_n(z) \cdot \sin \varphi \, dz \]

Fig. 2. Discharge integration and normal velocity distribution

For the discharge \( Q \) the mean normal velocity \( \bar{v}_n(z) \) averaged along the coordinate \( y \) is relevant. As only measured velocities at the \( z \)-positions of the acoustic paths are available, the integral (1) has to be solved numerically (2).

\[ Q \approx \frac{D}{2} \sum_{i=1}^{n} W_i \cdot \bar{v}_{ai} \cdot L_{wi} \cdot \sin \varphi \]

D is the conduit diameter, \( R \) is the radius, \( W_i \) the weighting coefficient, \( L_{wi} \) the path length, \( \varphi \) the path angle and \( n \) is the number of paths in one acoustic plane.

The weights \( W_i \) which are proposed in the IEC code [5] are determined on the basis of the Gauss-Jacobi method. The integration can be improved by calculating the weights according to the OWICS method [2]. This method is better adapted to the turbulent velocity distributions in circular pipes and therefore the weights are slightly different to the ones proposed in the appendix J of IEC41. All the simulation results presented in the following base on the OWICS method.

If the profile of the averaged normal velocity deviates from the perfect turbulent velocity distribution such as the disturbed distribution in Figure 2, left, an integration error results. The larger the number of paths is, the more measured information of the flow field is available and the better the integration gets.

1.2 Cross flow error

To understand how cross flow affects the measured velocities the superposition of the individual velocity components must be considered. Both, the single path and the crossed paths arrangement are treated in the following.

Single path

Figure 3 shows a path in a flow field with the axial component \( u \) and the cross flow velocity component \( v \). Both can vary in space with the coordinates \( x \), \( y \) and \( z \).

Fig. 3. Single path arrangement
For the transit time measurement the averaged projected velocity along a path is relevant. The transit times result from the sum or difference of the projected velocity \( \overline{v}_{pr} \) on the path and the sound velocity \( c \). For discharge integration, however, the axial velocity \( \overline{v}_a \) averaged along a path is of interest. This derived velocity is determined from the measured transit time downstream \( t_d \) and upstream \( t_u \) and the path angle \( \phi \). In order to distinguish between \( \overline{v}_a \) and the real axial velocity \( \overline{u} \), \( \overline{v}_a \) is denoted as derived velocity in the following.

\[
t_d = \frac{L_w}{c + \overline{v}_{pr}} \quad t_u = \frac{L_w}{c - \overline{v}_{pr}} \quad \overline{v}_a = \frac{\overline{v}_{pr}}{\cos \phi} = \frac{L_w}{2\cos \phi} \left( \frac{1}{t_d} - \frac{1}{t_u} \right) \tag{3} \tag{4}
\]

The derived velocity \( \overline{v}_a \) can be written as

\[
\overline{v}_a = \overline{u} + \overline{v} \cdot \tan \phi \tag{5}
\]

with

\[
\overline{u} = \frac{1}{L_w} \int_0^L u(p) \, dp \quad \overline{v} = \frac{1}{L_w} \int_0^L v(p) \, dp \tag{6}
\]

Whereas \( \overline{u} \) is the axial velocity and \( \overline{v} \) is the cross flow velocity, both averaged along the path (coordinate \( p \)). For the numerical integration of the discharge the normal velocity on the midplane averaged along the y-coordinate \( \overline{v}_n \) is needed, which in case of cross flow is not identical with \( \overline{v}_a \). For a single path arrangement the velocity \( \overline{v}_n \) is approximated by \( \overline{v}_a \) resulting in a cross flow error.

\[
\overline{v}_n \cong \overline{v}_a \tag{7}
\]

**Crossed paths**

To minimize the cross flow error two crossed paths usually are installed.

\[
\frac{1}{2} \left( \overline{v}_{aA} + \overline{v}_{aB} \right) = \frac{1}{2} \left[ \overline{u}_A + \overline{u}_B + \left( \overline{u}_A - \overline{u}_B \right) \cdot \tan \phi \right] \tag{9}
\]

Condition for zero cross flow error is that the derived path velocities are equal to the normal velocity

\[
\overline{v}_n = \frac{1}{2} \left( \overline{v}_{aA} + \overline{v}_{aB} \right) \tag{10}
\]
In reality with a disturbed flow profile this is not likely to be the case. The components \( \overline{u}_A \) and \( \overline{u}_B \), as well as \( \overline{v}_A \) and \( \overline{v}_B \), can differ as they are averaged velocities along different paths in a spatial velocity field. There is no error due to cross flow when the axial component \( u \) and the cross flow component \( v \) do not change along the pipe axis \( x \) within the measuring section.

For an analysis of the two separate errors, the integration error and the cross flow error, analytically determined, spatial velocity profiles will be used in the following.

## 2 Analytical flow profiles

### 2.1 Two dimensional profiles

Salami [3] uses an analytical description of two dimensional profiles to approximate a typical velocity distribution after a disturbance. An example of such a profile is given by

\[
v(r, \theta) = (1 - r)^{\frac{1}{2}} - \frac{0.5}{\pi} \cdot r \cdot (1 - r)^{\frac{3}{2}} \cdot \theta \cdot \sin \theta
\]

where \( \theta \) denotes the angle between 0 to \( 2\pi \). The radius of the circular section is set to 1. Figure 5 shows the contour plot of this profile.

![Contour plot of disturbed flow profile](image)

Voser [4] used these profiles to analyze the uncertainty of a four paths installation. He found that the integration error is a function of the installation angle \( \alpha \). In his analysis he assumed that the flow profile does not vary in direction of the pipe axis and accordingly that no cross flow occurs.

### 2.2 Three dimensional profiles

In the following flow profiles are considered which vary in direction of the pipe axis. It is assumed that disturbances will be damped out in a straight pipe and the disturbed velocity profile will turn into a symmetric turbulent profile within a certain distance. This distance depends on the Reynolds number and the wall roughness.

The formula (11) for the disturbed profile is composed of two terms. The first term is a simple approach to describe a turbulent profile. The exponent \( n \) depends on the Reynolds number and the wall roughness. For the following simulations \( n = 9 \) is assumed.

\[
v_{\text{turb}}(r) = (1 - r)^{\frac{1}{2}}
\]

The second term describes the disturbance.

\[
S(r, \theta) = -\frac{0.5}{\pi} \cdot r \cdot (1 - r)^{\frac{3}{2}} \cdot \theta \cdot \sin \theta.
\]

It is assumed that the influence of the disturbance term reduces exponentially along the pipe axis. Therefore a new variable \( s \) in the direction of the pipe axis is introduced.

\[
v(r, \theta, s) = C(s) \cdot v_{\text{turb}}(r) + K(s) \cdot S(r, \theta)
\]

whereas \( K(s) = e^{-5s} \).

The factor \( K(s) \) weights the disturbance term. For \( s = 1 \) the disturbance term becomes negligible. To fulfill the continuity equation in each section, the coefficient \( C(s) \) is introduced.

![Acoustic path](image)
Figure 6 shows the transition zone and the acoustic planes located within this zone. The development of the velocity profile for varying position \( s \) is shown in Figure 7. While the maximum velocity at \( s = 0 \) is in the bottom part, it is gradually moving to the center with increasing distance \( s \). Such a transition involves a cross flow which is dominantly in vertical direction.

\[
K(s) = e^{5s}
\]

\[
\theta = \sin(15.01) - 4^\theta
\]

\[
\theta = \sin(16813.01) - 4^\theta
\]

Fig. 6. Transition zone

Fig. 7. Development of fully developed turbulent velocity profile downstream of a disturbance

2.3 Results of the analysis

It is assumed that the transition length \( L \) is 10D. The chosen location of the ADM is at \( a = 2D \). For the analysis two profiles are investigated based on the following equations. Both, the horizontal and the vertical installation are tested to find the influence of the path orientation.

Disturbed velocity profile I \quad (eq.11) \quad Disturbed velocity profile II \quad (15)

\[
v(r, \theta) = (1 - r)\frac{1}{\pi} - \frac{0.5}{r} \cdot (1 - r)^\frac{1}{2} \cdot \theta \cdot \sin \theta
\]

\[
v(r, \theta) = (1 - r)^\frac{1}{2} + 0.6813 \cdot r \cdot (1 - r)^\frac{1}{2} \cdot e^{-0.10} \cdot \sin^2(\theta)
\]
Disturbed velocity profile I

Horizontal installation 
$\alpha = 0^\circ$

Vertical installation 
$\alpha = 90^\circ$

As expected the integration error decreases with an increasing number of paths. The installation angle has an important influence on the measuring uncertainty. The error of a single measuring plane (A or B) is much larger for the vertical installation than for the horizontal installation. This is due to the fact that the shift of the velocity maximum leads to cross flow in vertical direction. If cross flow acts in direction of the path orientation it affects the path velocities more.

Disturbed velocity profile II

Horizontal installation 
$\alpha = 0^\circ$

Vertical installation 
$\alpha = 90^\circ$

Fig. 8. Simulated error of the disturbed velocity profile I as a function of the path number in one plane [6]

Fig. 9. Simulated error of the disturbed velocity profile II [6]
For the horizontally installed paths the errors of a single measuring plane are identical for both planes, since the disturbed profile is almost symmetrical to the vertical axis. The deformation of the profile produces once more a vertical cross flow and has now influence on the horizontal paths, since a cross flow component which is perpendicular to the paths has no effect on the measured velocities. The transition of a disturbed velocity profile is just one possible cause for cross flow. Secondary flow caused by centrifugal forces in bends also cause velocity components perpendicular to the pipe axis. Very often swirling flows are observed in hydro power stations. Swirl or rotation of velocity profiles induces cross flow, too.

**Rotated velocity profile II**

In the following the disturbed profile II is additionally rotated to effect cross flow component. Therefore the rotation angle $\beta$ is introduced which is varied along the transition zone from $0^\circ$ to $360^\circ$. So, the profile experiences one full rotation within the transition zone.

$s = 0, \beta = 0^\circ$

$s = 0.2, \beta = 72^\circ$

$s = 0.4, \beta = 144^\circ$

$s = 0.6, \beta = 216^\circ$

$s = 0.8, \beta = 288^\circ$

$s = 1, \beta = 360^\circ$

**Fig. 10. Development of velocity profile from disturbed into turbulent with a rotation**

For this simulation even with a large number of crossed paths an error remains. This simulated error composes of the cross flow error and the integration error. To obtain the integration error, the normal velocity $v_n$ averaged along the coordinate y on the mid-plane is used for the numerical integration of the discharge. To obtain the part of the cross flow error the integration error is subtracted from the simulation error

$$e_{cross} = e_{sim} - e_{int}.$$ 

**Fig. 11. Errors in the rotating flow profile**

Only the integration error can be reduced applying an increased number of paths. The cross flow error remains constant as it is a result of the spatial variation of the velocity within the measuring section.

In real flow profiles the influence of cross flow can be larger. In the next chapter a consideration of the integration error and the cross flow error is made on the basis of a CFD simulation (computational fluid dynamics).
3 CFD simulated profile

For a further uncertainty analysis the CFD simulated flow profile of the penstock from the Three Gorges Dam in China is used. If an ADM is installed, e.g. at the turbine inlet, the flow will be disturbed from two upstream bends and the downstream spiral casing.

![Fig. 12. Location of a simulated measuring section of the Three Gorges penstock](image)

A measuring section with two crossed planes and a path angle of 70° is assumed.

![Fig. 13. Flow field visualization with the absolute velocity on the left picture and the gradient of the cross flow velocity in axial direction x on the right side](image)

On Figure 13, left, a slight change of the velocity in the measuring section is observed. The gradient of the cross flow, Figure 13, right, demonstrates that the cross flow component varies along the pipe axis within the measuring section.
If we consider just two paths for illustration of the cross flow error, we see that the averaged path velocities of path A and path B results to 7.76 m/s. The normal velocity $\overline{V}_n$ averaged along the (black) line perpendicular to the pipe axis is 7.84 m/s. This shows that the approximation of $\overline{V}_n$ by the averaging the two path velocities $\overline{v}_{aA}$ and $\overline{v}_{aB}$ is inaccurate due to cross flow.

A 2x4 path and a 2x8 path installation are tested in this simulated flow profile. Calculating the averaged velocity on each path from the simulated flow profile the discharge can be integrated and compared to the reference discharge. The resulting deviation from the reference flow (simulated error) is due to the cross flow error and the integration error. The integration error can be reduced with the number of paths. To determine the cross flow error the integration error is again subtracted from the simulation error.

In table 1 the individual errors are listed. Obviously the integration error is reduced to almost zero for the 2x8 path installation. In contrast, it is important to note that the cross flow error remains constant as it is a result from the spatial variation of the flow field within the measuring section.

<table>
<thead>
<tr>
<th>Installation</th>
<th>Simulated error $e_{sim}$</th>
<th>Integration error $e_{int}$</th>
<th>Cross flow error $e_{cross}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2x4 path</td>
<td>-0.82%</td>
<td>-0.25%</td>
<td>-0.57%</td>
</tr>
<tr>
<td>2x8 path</td>
<td>-0.52%</td>
<td>0.04%</td>
<td>-0.56%</td>
</tr>
</tbody>
</table>

*Table 1. Errors of the analysis using CFD simulation*

4 Conclusion

The analysis with analytical, spatially varying profiles shows that the cross flow error using just single paths can be large. An orientation of the paths perpendicular to the cross flow component reduces the cross flow error.

With a crossed paths arrangement the error arising from cross flow can be reduced to a minimum, if no swirl or rotation is present. The integration error decreases with a higher number of paths. In case of swirl or rotation an additional cross flow error is found which cannot be reduced by an increased number of acoustic paths.

This finding is confirmed by a CFD simulation of the penstock flow of the Three Gorges Dam. The occurring complex spatial flow profile and the varying cross flow within the measuring section is responsible for a remaining cross flow error even with crossed paths arrangement. Whereas the integration error can be reduced using an increased number of paths, the cross flow error remains for this example constant at about 0.6%. Thus, the cross flow error can lead for unfavorable installations to a systematical error of the ADM.
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