Numerically calculated rotor dynamic coefficients of a pump rotor side space

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ABSTRACT

A procedure was tested to determine rotor dynamic coefficients on the basis of numerical, three-dimensional flow simulations in the labyrinth seals and the rotor side space of a pump using a commercial CFD-code. Choosing an adequate, rotating co-ordinate system made steady calculations possible. Rotor dynamic coefficients could be identified from the regression of calculated forces.

A test case of plain seal configurations in combination with an intermediate chamber, for which detailed experimental data were available, was used to verify the CFD results. On the one hand simplified calculations of a plain seal configuration were performed, on the other hand three-dimensional calculations were done on the entire test case geometry. The agreement of numerically determined rotor dynamic coefficients with the experimental data and with the bulk-flow theory of Childs proved to be good. Furthermore, a three-dimensional flow simulation of an entire pump rotor side space was performed, demonstrating that the proposed method for calculation of rotor dynamic coefficients can be applied to very general cases.

Keywords: turbomachinery seals, rotor dynamic coefficients, pump rotor side space, CFD calculations, rotating co-ordinate system

1. ROTOR DYNAMIC FORCES

Any movement of the axis of rotation of the impeller-shaft relative to its casing induces fluid forces on the shaft and the casing, which in turn increase or decrease the rotor deflection or vibration. Contributions to these rotor dynamic forces can arise from seals, the rotor side space, the flow through the impeller, leakage flows, or the flow in the bearings themselves [1].

When designing high speed turbomachines the resulting fluid forces and rotor deflection must be known to begin with. For this reason rotor dynamic coefficients are required as input data for the prediction of rotor vibration. In general the dynamic behavior of the rotor system is described by linear models with time invariant system parameters in terms
of differential equations, expressing the dynamic equilibrium of inertia, damping, stiffness and external forces [2].

Assuming a linear relationship of force and displacement and neglecting influences of high derivatives of the motion, this force-displacement model may be described with the following rotor dynamic stiffness, damping and mass matrices:

\[
\begin{bmatrix}
-F_x \\
-F_y
\end{bmatrix} =
\begin{bmatrix}
K_{xx} & K_{yx} \\
-K_{yx} & K_{yy}
\end{bmatrix}\begin{bmatrix}x\end{bmatrix} +
\begin{bmatrix}
D_{xx} & D_{yx} \\
-D_{yx} & D_{yy}
\end{bmatrix}\begin{bmatrix}\dot{x}\end{bmatrix} +
\begin{bmatrix}
M_{xx} & M_{yx} \\
-M_{yx} & M_{yy}
\end{bmatrix}\begin{bmatrix}\ddot{x}\end{bmatrix} \quad (1)
\]

For small motion around a centered position the cross-coupled terms of the damping and stiffness matrices become equal in magnitude based on their rotational symmetry [2]. According to experimental findings the cross-coupled inertia terms of the mass matrix may be neglected and are set to be zero. However, the direct inertia term \(M\) cannot be neglected except in cases where laminar seal flow dominates, e.g. for slide bearings. Thus, the coefficient matrices can be simplified in the case of small concentric perturbations as follows:

\[
\begin{bmatrix}
-F_x \\
-F_y
\end{bmatrix} =
\begin{bmatrix}
K_{xx} & K_{xc} \\
-K_{xc} & K_c
\end{bmatrix}\begin{bmatrix}x\end{bmatrix} +
\begin{bmatrix}
D_{xx} & D_{xc} \\
-D_{xc} & D_c
\end{bmatrix}\begin{bmatrix}\dot{x}\end{bmatrix} +
\begin{bmatrix}
M & 0 \\
0 & M
\end{bmatrix}\begin{bmatrix}\ddot{x}\end{bmatrix} \quad (2)
\]

Labyrinth seals are often the major source for radial fluid forces in turbomachinery and may, in the worst case, destabilize their rotors. On the other hand, the seals are the key elements controlling the leakage flow of turbomachines. Optimizing leakage flow and friction will contribute to improving the efficiency of the machines. In practice one has to find a compromise between the often conflicting demands arising from efficiency improvement and rotor dynamic stability.

For seals the coefficient matrices are independent of the type of rotor motion but strongly dependent upon the operational conditions of the seal, such as the leakage flow or the rotor speed and also depend largely on the preswirl of the flow entering the seal. These particular effects were systematically investigated in a series of research work at the Swiss Federal Institute Zurich, ETHZ [3-6]. On the basis of experimentally measured forces under varying operational conditions of the seal and rotor motion the stiffness, damping and mass matrices of equation (2) could be determined. Below, selected results of these studies will be used as a reference for comparison with the results of the study being presented.

To calculate the rotor dynamic behavior of turbomachinery seals well-validated tools, such as the bulk-flow theory of Childs [2], are available. The difficulty in applying these tools very often lies in making a preliminary estimate of the flow conditions into the seal part. To avoid this difficulty a procedure was tested to determine rotor dynamic coefficients on the basis of numerical flow calculations using a commercial CFD code. Choosing an adequate, rotating co-ordinate system, rotor dynamic coefficients could be determined
directly through the integration of calculated pressure distributions, as will be shown below. This procedure is also described by Moore and Palazzolo [10].

2. IDENTIFICATION OF COEFFICIENTS

To determine rotor dynamic coefficients a procedure identical to the one being used in the experimental studies was chosen in the numerical simulations. The experiments on the test rig described in [4], [5], [6] were performed with a circular orbit of the rotor eccentricity, which can be described with the following harmonic functions:

\[ x = \varepsilon S_0 \cos(\Omega_E t), \quad y = \varepsilon S_0 \sin(\Omega_E t), \quad \varphi = \Omega_E t \quad (3) \]

Fig. 1 Concentrical orbital shaft motion in the absolute and the relative co-ordinate system [5] (non-synchronous orbital motion of angular frequency $\Omega_E$ and relative amplitude $\varepsilon$)

Fig. 2 Schematic representation of the rotor dynamic coefficients in the force vector diagram
The orbital motion causes an unsteady flow in the seal as indicated in Figure 1 (left). Unsteadiness of the flow, however, increases the computational expense for CFD calculations immensely. To avoid this problem, a rotating co-ordinate system was chosen for the present study, in which the flow becomes steady. Figure 1 (right), shows this relative co-ordinate system with a relative rotor angular velocity of \( \Omega_R = \Omega_P - \Omega_E \). Accordingly the stator must rotate backwards with a relative angular velocity of \( \Omega_S = -\Omega_E \). In the absolute co-ordinate system this corresponds of course to an orbit motion equivalent to equation (3).

With respect to the orbit center a radial force \( F_r \) and circumferential force \( F_c \) acting on the rotor (Figure 1) can be determined by integrating the pressure distributions. In the chosen relative system the integration of the pressure distribution along the y-axis yields the force component \( F_r \) and the integration along the x-axis yields \( F_c \):

\[
F_r = F_x = -\int_0^{2\pi} p_{\text{Rotor}} \cos \phi R d\phi dy \\
F_c = F_y = -\int_0^{2\pi} p_{\text{Rotor}} \sin \phi R d\phi dy
\]  

(4), (5)

These integrations were performed for constant leakage, rotor speed, and preswirl at the inlet \( (Re_m = c_m^2 S_0 \mathcal{N} = \text{const}; u/c_m = \text{const}; c_{\text{ax}}/c_m = \text{const}) \) as a function of the orbit angular frequency \( \Omega_E \). Introducing these forces into equations (2) and setting the time to \( t = 0 \) in equation (3) leads to:

\[
\frac{F_r(\Omega_E(t = 0))}{\varepsilon S_0} = \frac{F_c(\Omega_E)}{\varepsilon S_0} = -K - D \Omega_E + M \Omega_E^2
\]  

(6)

\[
\frac{F_r(\Omega_E(t = 0))}{\varepsilon S_0} = \frac{F_c(\Omega_E)}{\varepsilon S_0} = K_r - D \Omega_E
\]  

(7)

In order to determine the rotor dynamic coefficients, \( F_r \) and \( F_c \) have to be determined as a function of the orbit angular frequency \( \Omega_E \) as follows from equation (6) and (7). For numerical calculations this means that solely the angular velocity of stator and rotor have to be varied in the relative co-ordinate system. Employing a second order regression on equation (6), the coefficients of stiffness \( K \), cross damping \( D \), and inertia \( M \) can be determined from the calculated radial forces \( F_r = f(\Omega_E) \). The damping coefficient \( D \) and the cross stiffness coefficient \( K_c \) are determined from linear regression of \( F_c = f(\Omega_E) \) in equation (7). Iwatsubo [7] describes in detail the procedures for identification of the coefficient matrices with this method. An example for \( F_r = f(\Omega_E) \) and \( F_c = f(\Omega_E) \) is given in paragraph 4, Figure 6.

3. CFD CALCULATIONS

Numerical simulation of the flow was performed using the commercial CFD code, namely CFX-TASCflow, version 2.10. A cluster of Sun Ultra 80 workstations, each with 2x450 MHz CPUs and 2 Gbyte RAM, was utilized. Computations were performed in parallel. For grid generation we employed ICEM Hexa, version 4.1.
For the calculations presented here very fine grids close to the walls were chosen due to the nature of the problem. In order to avoid inconsistencies in the employed log law wall functions, it is assumed that the surface coincides with the edge of the viscous sublayer, which is defined by \( y^+ = \Delta n \frac{u_c}{N} = 11 \). This is the intersection between the logarithmic and the linear near wall profile. The computed \( y^+ \) is not allowed to fall below this limit when the "fixed \( y^+ \)" option in CFX-TASCflow is activated [8]. Therefore, all grid points are outside the viscous sublayer and the wall shear stress is correctly computed.

Overall, three series of calculations where performed, all closely related to the geometry of the ETHZ test case [5] which was used for validation of the results. The geometry of this case is shown in Figure 3.

**Plain Seal**

The first series of calculations dealt only with the second plain seal of the test case. For the full three-dimensional simulation of the seal flow a number of \( 10^5 \) elements were used for discretization. The 60 case studies needed for identification of the rotor dynamic coefficients where performed by the student B. Simperl and are described in his diploma thesis [9]. The inlet losses to the seal for a given inlet swirl component where modeled with a so-called porous region which allowed insertion of source terms in the CFD calculations to simulate the inlet pressure losses. The inlet swirl was assumed to be \( c_{\text{in}}/u = 0.25 \) and the inlet loss coefficient modeled within the porous region was \( \zeta_L = 0.5 \). The total pressure was chosen to be constant at the inlet, allowing circumferentially varying inlet...
flow. The rotor angular speed was varied in order to determine rotor dynamic coefficients. These simulations were performed for three different values of leakage flow.

**Full Test Case Simulation**

A total number of $8 \cdot 10^5$ elements have been used for discretization of the full test case. These simulations provided a lot of insight into the detailed flow effects occurring in this stepped labyrinth seal with an intermediate chamber. In the chamber, jet separation and reattachment as reported by [6] could be observed. Figure 5 displays the circumferentially extended, counter-rotating vortices induced in the labyrinth chamber in meridional projection. For comparison with the plain seal calculation the pressure distributions of the second seal were evaluated and forces and coefficients extracted. To simulate the full test case the introduction of the porous region for loss modeling was no longer necessary. Figure 6 shows the correspondence of the calculated forces on the basis of the plain seal simulation with the full test case simulation.

**Rotor Side Space Simulation**

In a third calculation a full rotor side space including the labyrinth seal section was calculated. In order to provide reliable results, the number of elements had to be increased to $1.8 \cdot 10^6$. The accuracy of the results was tested with a well validated preliminary two-dimensional calculation, e.g. the friction coefficient corresponded with the two-dimensional calculation within 1 percent. With the full three-dimensional simulation the forces emerging from the individual sections of the rotor side space can be compared in magnitude and direction, Figure 8. In contrast to the first two methods of simulation, the inlet flow to the seal, which is generally the major source of uncertainty when performing conventional rotor dynamic seal calculations, did not need to be assumed.

![Fig. 5 Vortex flow in the labyrinth chamber with the entering jet being attached to the chamber wall on half the periphery and being detached on the other half (streamline projection on the meridian plane)](image-url)
4. RESULTS

Figure 6 displays a typical set of data which were used to determine the rotor dynamic coefficients for given boundary conditions (leakage flow, rotor speed, preswirl at the inlet). The general features, linear decrease of the circumferential force $F_c$ and parabolic behavior of the radial force $F_r$, correspond well with the experimental findings [4]. Accordingly, the damping coefficient $D$ and the cross stiffness coefficient $K_c$ are determined from the linear regression of $F_c = f(\Omega_E)$. Applying a second order regression to $F_r = f(\Omega_E)$, the coefficients of stiffness $K$, cross damping $D_c$, and inertia $M$ can be determined.

Included in this graph are the calculations of the second plain seal, modeled with a porous inlet region (■), and of the full test case simulation (▲). The agreement of the two types of numerical calculations is good even when the inlet conditions of the plain seal calculation kept constant at $\zeta_E = 0.5$ and $c_{uin}/u = 0.25$.

From a series of calculations, such as those presented in Figure 6, rotor dynamic coefficients could be identified for variable rotor circumferential speed $u$ and three different leakage flow rates. The results of the plain seal and the full test case simulation are included in Figure 7 and are compared to results from bulk-flow calculations [2] as well as to experimental data [3]. For reasons of clarity only experimental data at $c_m = 8.3$ m/s are included in the graph. Obviously the experimental data suffer from an important scatter, but reproduce the same trends and order of magnitudes as the simulated data. The numerical calculations correspond well with the bulk-flow results. Regarding the inertia data (coefficient $M$) the trend towards higher values observed in the experiment seems to be confirmed.

Since the comparison with the ETHZ test case showed good results, an example of a full pump rotor side space was simulated. Figure 8 compares the force components of the different sections of the rotor side space. The chamber between the two plain seals contributes only little to the total force, its role lies in conditioning the flow to the second seal. Rather surprisingly, the rotor side space force $F_{rss}$ is large, although the circumferential, relative gap variation is very small in this section. It can be presumed that this influence is often underestimated in the rotor dynamic design process. The total force acts upwards in this example, in direction of the displacement velocity $\dot{y}$. Its angle lies in a range between $0 < \phi < 180$ degrees, and accordingly greatly destabilizes the rotor. This destabilization is caused by the inlet swirl which is not reduced in the rotor side space. At the inlet to the seal $c_{uin}/c_m$ is larger than 1.
Fig. 6 Variation of the radial and circumferential forces with the orbit frequency for constant leakage flow ($c_m = 8.3 \, \text{m/s}$), rotor speed ($\Omega_R = 121 \, \text{1/s}$), and preswirl at the inlet (porous inlet region (■), full test case simulation (▲)).

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Fig. 7 Comparison of coefficients determined with bulk-flow theory [2], with CFD (plain seal and full test case simulation), and with experiments [6].
5. CONCLUSIONS

It was demonstrated that a commercial CFD code could be used to calculate rotor dynamic coefficients of a pump rotor side space and seals. The procedure is straightforward simple to apply, especially if CFD tools and CFD-experienced people are involved in the design process of the pump anyway. Correct inflow data are needed for good prediction of rotor dynamic coefficients of seals and other sections relevant to rotor dynamics - data which can be provided by numerical calculations.

Rotor dynamic coefficients were predicted by choosing an adequate, rotating co-ordinate system with steady flow conditions, in spite of the unsteady nature of the problem. In this relative system pressure distributions could be determined from which the forces were integrated. In a next step rotor dynamic coefficients were determined from the regression of radial and circumferential forces, calculated as a function of the angular frequency of the rotor center orbit motion.

The procedure was validated on the basis of a test case of plain seal configurations in combination with an intermediate chamber. On the one hand, simplified calculations of a plain seal configuration were performed, on the other hand three-dimensional calculations were applied to the entire test case geometry. The numerically determined rotor dynamic coefficients corresponded with the experimental data and with the bulk-flow theory of Childs.
To show that the procedure may be generalized, a three-dimensional flow simulation of an entire pump rotor side space was performed. A test demonstrated that highly destabilizing forces may arise in the rotor side space. The computational expense of such calculations is still very high but certainly small in comparison to experimental methods of determining rotor dynamic coefficients.

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7. REFERENCES


7. NOMENCLATURE

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